# Observer Based Hybrid Intelligent Scheme for Activated Sludge Wastewater Treatment

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***Abstract*—** This paper presents a hybrid intelligent system to control the substrate/biomass concentration ratio in an activated sludge wastewater treatment process; substrate and biomass are estimated by an observer, which is based on a discrete-time high order neural network (RHONN) trained on-line with an extended Kalman filter (EKF) based algorithm. The intelligent system and neural observer performance is illustrated via simulations; different external disturbance scenarios are provided to show the robustness of the proposed scheme.

## I. Introduction

Recently, the use of wastewater treatment plants (WWTP) has increased due to environmental issues. Regarding these facts, wastewater treatment plants based on activated sludge technology are being used for urban wastewater.

Controlling a WWTP is a difficult task, and different strategies should be implemented in order to maintain good operations conditions in presence of external disturbances. Several authors have proposed different control schemes (Sanchez *et. al.*, 2004), (Olsson *et al.,* 1999), (Vera *et al.,* 2003), and (Lakrori *et al.,* 1989). These approaches make an effort to keep the quality of the effluent, while others try to minimize energy (Tong *et al.,* 1980).

However, the application of these strategies requires sensors allowing the measurements of the process main variables; these sensors could be very expensive and require elaborated maintenance procedures. Due to these facts, state estimation applied to WWTP and to biological processes has received special attention by many authors, who have obtained interesting results in different directions and for different purposes. Most of the existing results need the use of a special nonlinear transformation (Lopez *et al.,* 2004). Other kind of observers are those called robust, which have good performance under uncertainties although their design is too complex and has very restrictive conditions (Alcaraz *et al.,* 2007).

The main problem of all the approaches mentioned above is the requirement to know at least partially the plant dynamic model. However, other kinds of observers have been recently proposed: neural observers (Pozniak *et al.,* 2001), (Sanchez *et al.,* 2007), (Rovhitakis *et al.,* 2000) do not need this requirement.

This paper is related to two main aspects: 1) the development of the recurrent high order neural observer (RHONO) (Sanchez *et al.,* 2007), based on a RHONN which is applied to a WWTP in order to estimate the substrate and biomass concentration using dissolved oxygen as the measured variable. The neural observer learning uses and extended Kalman filter algorithm (Sanchez *et. al.*, 2004). 2) Furthermore, we associate this neural observer to an intelligent control, which uses fuzzy logic to regulate the substrate/biomass concentration ratio, in order to ensure effluent quality even in presence of external disturbances.

## II. Process Description

The process of a typical aerobic treatment plant corresponds to the benchmark of the European group COST 624 (Beteau *et. al.*, 2000); the diagram of the aerobic treatment plant is presented in Fig. 1. The two main parts are: the bioreactor which usually can be modeled by five perfectly mixed tanks and the settler modeled with 10 layers (Takacs *et. al.*, 1991).

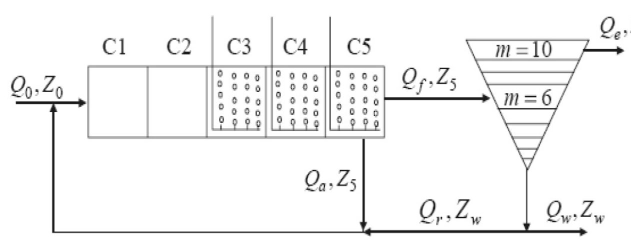



Fig.1. Activated Sludge Wastewater Process Scheme.

The first two compartments of the bioreactor are non-aerated and there denitrification takes place; the next three compartments (nitrification process) are aerated. $Q_0$ and $Z_0$ are respectively the flow rate and the concentrations of the plant influent (disturbances); $Q_f$ and $Z_5$ are the flow rate and concentration at the bioreactor output; $Q_e$ and $Z_e$ are the flow rate and concentration of the plant effluent; $Q_w$ and $Z_w$ are the flow rate and concentration of the sludge wastage; $Q_r$ is the external recycle flow rate and $Q_a$ is the internal recycle flow rate.

The global mathematical model for this process requires 145 nonlinear differential equations, obtained by calculating mass balances for the five sections of the bioreactor and the 10 layers of the settler. Benchmark simulations are implemented with the simulator provided by (Beteau *et. al.*, 2000). All the flow rates used in the model are in $m^3$/day. The process model uses 13 state variables according with ASM1 (Henze *et. al.*, 1986). The main variables are

$S_S$ Fast biodegradable substrate.
$X_{BH}$ Active heterotropic biomass.
$X_{BA}$ Active autotrofic biomass.
$S_O$ Dissolved oxygen.
$S_{NO}$ Nitrate and nitrite nitrogen.
$S_{NH}$ Amoniacal nitrogen.

## III. Discrete-Time Recurrent High Order Neural Network

Let consider a MIMO nonlinear system

$$x_i(k+1) = F(x(k), u(k)) \tag{1}$$

where $x \in \Re^n$, $u \in \Re^m$ and $F \in \Re^n \times \Re^m \to \Re^n$ is a nonlinear function. Now a discrete-time recurrent high order neural network (RHONN) can be presented as:

$$x_i(k+1) = w_i^T z_i(x(k), u(k)),\ i = 1, \ldots, n \tag{2}$$

where $x_i$ ($i$ = 1,2,…,$n$) is the state of the $i$th neuron, $L_i$ is the respective number of higher-order connections, $n$ is the state dimension, $\{I_1, I_2, \ldots, I_{Li}\}$ is a collection of non-ordered subsets of {1,2,…,$n$}, $w_i$ ($i$ = 1,2,…,$n$) is the respective on-line adapted weight vector, and $z_i(x(k), u(k))$ is given by

$$z_i(x(k), u(k)) = \begin{bmatrix} z_{i_1} \\ z_{i_2} \\ \vdots \\ z_{i_{Li}} \end{bmatrix} = \begin{bmatrix} \Pi_{j \in I_1} y_i^{di_j(1)} \\ \Pi_{j \in I_2} y_i^{di_j(2)} \\ \vdots \\ \Pi_{j \in I_{L_i}} y_i^{di_j(L_i)} \end{bmatrix} \tag{3}$$

with $d_j(k)$ nonnegative integers and $y_i$ defined as follows:

$$y_i = \begin{bmatrix} y_{i_1} \\ \vdots \\ y_{i_1} \\ y_{i_{n+1}} \\ \vdots \\ y_{i_{n+m}} \end{bmatrix} = \begin{bmatrix} S(x_1) \\ \vdots \\ S(x_n) \\ u_1 \\ \vdots \\ u_m \end{bmatrix} \tag{4}$$

In (4), $u = [u_1, u_2, \ldots, u_m]^T$ is the input vector to the neural network (NN), and $S(\bullet)$ is defined by

$$S(x) = \frac{1}{1 + \exp(-\beta x)} + \in \tag{5}$$

We consider now the problem to approximate the general-time nonlinear system (1), by the following discrete-time RHONN (Ricalde *et. al.*, 2005):

$$\chi_i(k+1) = w_i^{*T} z_i(x(k), u(k)) + \in_{z_i},\ i = 1, \ldots, n \tag{6}$$

where $\chi_i$ is the $i$th plant state, $\epsilon_z$ is the bounded approximation error, which can be reduced by increasing the number of adjustable weights (Rovhitakis *et al.,* 2000). Assume that there exists ideal weight vector $w_i^*$ such that $\|\epsilon_z\|$ can be minimized on a compact set $\Omega_{Z_i} \subset \Re^{L_i}$. In general, it is assumed that this vector exists and is constant but unknown. Let us define its estimate as $w_i$ and the estimation error as

$$\tilde{w}_i(k) = w_i^* - w_i(k) \tag{7}$$

### A. The EKF Training algorithm

For KF-based neural network training, the network weights become the states to be estimated. In this case, the error between the neural network output and the measured plant output can be considered as additive white noise. Due to the fact that the neural network mapping is nonlinear, an EKF-type is required. The training goal is to find the optimal weight values which minimize the prediction error (Sanchez *et al.,* 2004), (Song *et al.,* 2005). In this work, we use an EKF-based training algorithm described by

$$\begin{aligned} w_i(k+1) &= w_i(k) + \eta_i K_i(k) e_i(K) \\ K_i(k) &= P_i(k) H_i(k) M_i(k) \qquad i = 1, \ldots, n \\ P_i(k+1) &= P_i(k) - K_i(k) H_i^T(k) P_i(k) + Q_i(k) \end{aligned} \tag{8}$$

with

$$\begin{aligned} M_i(k) &= [R_i(k) + H_i^T(k) P_i(k) H_i(k)]^{-1} \\ e_i(k) &= y(k) - \hat{y}(k) \end{aligned} \tag{9}$$

where $e(k) \in \Re^p$ is the observation error and $P_i(k) \in \Re^{L_i \times L_i}$ is the weight estimation error covariance matrix at step $k$, $w_i \in \Re^{L_i}$ is the weight (state) vector, $L_i$ is the respective number neural network weights, $y \in \Re^p$ is the plant output, $\tilde{y} \in \Re^p$ is the NN output, $n$ is the number of states, $K_i \in \Re^{L_i \times p}$ is the Kalman gain matrix, $Q_i \in \Re^{L_i \times L_i}$ is the NN weight estimation noise covariance matrix, $R_i \in \Re^{p \times p}$ is the error noise covariance, and $H_i \in \Re^{L_i \times p}$ is a matrix, in which each entry ($H_{ij}$) is the derivative of the *i-th* neural output with respect to *ij*-th NN weight, ($w_{ij}$), given as follows:

$$H_{ij}(k) = \left[ \frac{\partial \hat{y}(k)}{\partial w_{ij}(k)} \right]^T \quad (10)$$

where $i$=1,…,$n$ and $j$=1,…,$L_i$. Usually $P_i$ and $Q_i$ are initialized as diagonal matrices, with entries $P_i(0)$ and $Q_i(0)$, respectively. It is important to remark that $H_i(k)$, $K_i(k)$ and $P_i(k)$ for the EKF are bounded; for a detailed explanation of this fact see (Hamed *et al.,* 2004). To obtain $H$ in (10) is not an easy task. In this case $\hat{y}(k) = x_i(k)$, so by the chain rule, we have

$$\frac{\partial \hat{y}(k)}{\partial w_{ij}(k)} = \frac{\partial \hat{y}(k)}{\partial x_i(k)} \frac{\partial x_i(k)}{\partial w_{ij}(k)} \quad (11)$$

*B. Discrete Time Neural Observer*

In this section, we briefly present the neural observer, proposed in (Sanchez *et al.,* 2007). We consider the state of a discrete-time nonlinear system, which is assumed to be observable, given by

$$\begin{aligned} x(k+1) &= F(x(k), u(k)) + d(k) \\ y(k) &= Cx(k) \end{aligned} \quad (12)$$

where $x \in \Re^n$ is the state vector of the system, $u \in \Re^m$ is the input vector, $y(k) \in \Re^p$ is the output vector, $C \in \Re^{p \times n}$ is a known output matrix, $d(k) \in \Re^n$ is a disturbance vector and $F(\bullet)$ is a smooth vector field and $F_i(\bullet)$ its entries; hence (12) can be rewritten as:

$$\begin{aligned} x(k+1) &= [x_1(k) \dots x_i(k) \dots x_n(k)]^T \\ d(k) &= [d_1(k) \dots d_i(k) \dots d_n(k)]^T \\ x_i(k+1) &= F_i(x(k), u(k)) + d_i(k), \quad i = 1, \dots, n \\ y(k) &= Cx(k) \end{aligned} \quad (13)$$

For system (13), a Luenberger neural observer (RHONO) is proposed in (Sanchez *et al.,* 2007), with the following structure:

$$\begin{aligned} \hat{x}(k) &= [\hat{x}_1(k) \dots \hat{x}_i(k) \dots \hat{x}_n(k)]^T \\ \hat{x}_i(k+1) &= F_i(\hat{x}(k), u(k)) + L_i e(k) \\ \hat{y}(k) &= C\hat{x}(k), \qquad i = 1, \dots, n \end{aligned} \quad (14)$$

with $L_i \in \Re^p$, $w_i$ and $z_i$ as in (2); the weight vectors are updated on-line with a decoupled EKF (8)-(11). The output error is defined by

$$e(k) = y(k) - \tilde{y}(k) \quad (15)$$

and the state estimation error as

$$\tilde{x}(k) = x(k) - \hat{x}(k) \quad (16)$$

Hence the dynamic of (16) can be expressed as

$$\begin{aligned} \tilde{x}(k+1) &= x_i(k+1) - \hat{x}_i(k+1) \\ &= w_i^{*T} z_i(x(k), u(k)) + \epsilon_{z_i} + d_i(k) \\ &\quad - w_i^T z_i(\hat{x}(k), u(k)) - L_i e(k) \\ &= \tilde{w}_i z_i(\hat{x}(k), u(k)) + \epsilon_{z_i} + d_i(k) - L_i e(k) \end{aligned} \quad (17)$$

Considering (14) and (15)

$$e(k) = C\tilde{x}(k) \quad (18)$$

Then the error of (9) can be rewritten as

$$\tilde{x}(k+1) = \tilde{w}_i z_i(\hat{x}(k), u(k)) + \epsilon'_{z_i} - L_i C\tilde{x}(k) \quad (19)$$

where $\epsilon'_{Zi} = \epsilon_{Zi} + d_i(k)$. On the other hand the dynamics of (9) is

$$\tilde{w}_i(k+1) = w_i^* - w_i(k+1) = \tilde{w}_i(k) - \eta_i K_i(k) e(k) \quad (20)$$

For a detailed explanation of the synthesis and analysis of the neural observer see (Sanchez *et al.,* 2007).

*C. Observer Application*

To this end, the neural observer is applied to a WWTP as presented in subsection B, whose nonlinear dynamics is considered unknown. To estimate substrate and biomass concentrations with oxygen concentration measurement in the fifth compartment of the bioreactor, we use the RHONO (13) with $n$=3. The neural network used, for this state estimation, is given by

$$\begin{aligned} \hat{x}_1(k+1) &= w_{11}S(\hat{x}_1) + w_{12}S(\hat{x}_1)^2 S(\hat{x}_2)S(\hat{x}_3) + w_{13}S(\hat{x}_3) \\ &\quad + w_{14}S(u_1) \\ \hat{x}_2(k+1) &= w_{21}S(\hat{x}_2) + w_{22}S(\hat{x}_1)^2 S(\hat{x}_2)S(\hat{x}_3)^3 + w_{23}S(\hat{x}_2) \\ &\quad + w_{24}u_2 \\ \hat{x}_3(k+1) &= w_{31}S^2(\hat{x}_3) + w_{32}S(\hat{x}_1)S^2(\hat{x}_2)S(\hat{x}_3)^3 + w_{33}S(\hat{x}_3) \\ &\quad + w_{34}S(u_3) \\ \hat{y} &= \hat{x}_3 \end{aligned}$$

where $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ are the estimation of the fast biodegradable substrate ($S_S$), active heterotropic biomass ($X_{BH}$) and oxygen ($S_O$), respectively. The input $u_1$ is the flow rate of the plant influent $Q_0$, $u_2$ is the external recycle flow rate $Q_r$, and $u_3$ is the control action of oxygen in the fifth compartment of the bioreactor.

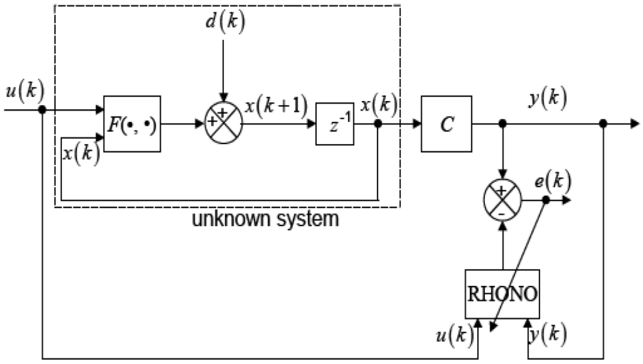
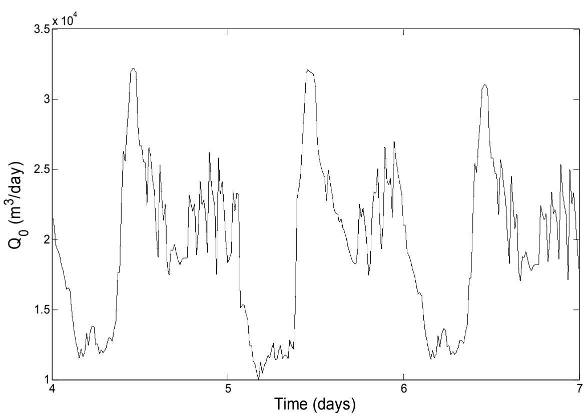


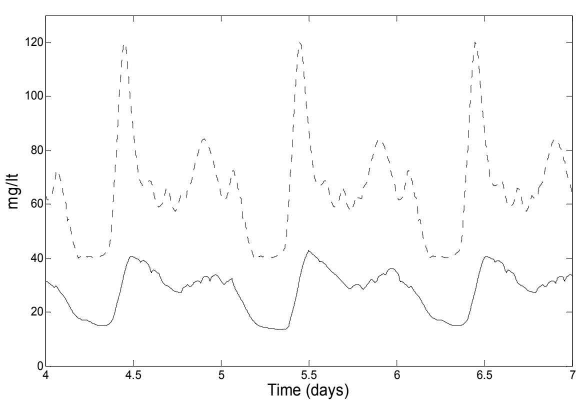

Fig.2. Observation scheme.

The training is performed on-line, using a parallel configuration as displayed in Fig.2. All the NN states are initialized randomly. The covariance matrices are initialized as diagonal, with nonzero elements as: $P_i(0) = 800000$, $Q_i(0) = 200$ and $R_i(0) = 4000$, ($i$=1,2,3), respectively.

### *D. Simulation Results*




(a)




(b)

Fig.3. External Disturbance: (a) Volumetric flux influent. (b) Fast biodegradable substrate (solid line) and heterotropic biomass concentrations influent (dashed line).

For simulation, the scenario considered is: the first four days a constant disturbance is included; finally for the next days a time variable disturbance is incepted, as displayed in Fig.3.

As can be seen in Fig. 4, there is exact convergence for the state variable ($S_o$) as expected, whereas for the other state variables good estimations are obtained, see Fig.5.

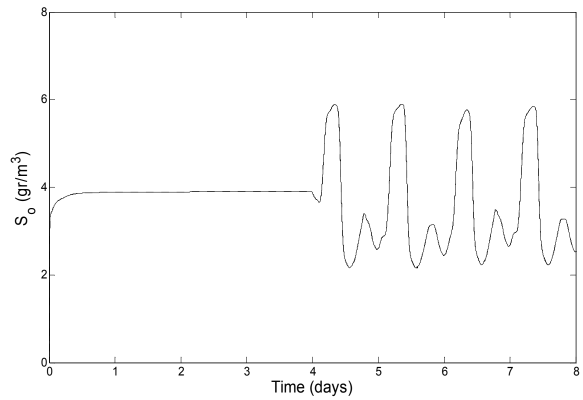



Fig.4. Oxygen concentrations (solid line) and their respective estimates (dashed line).

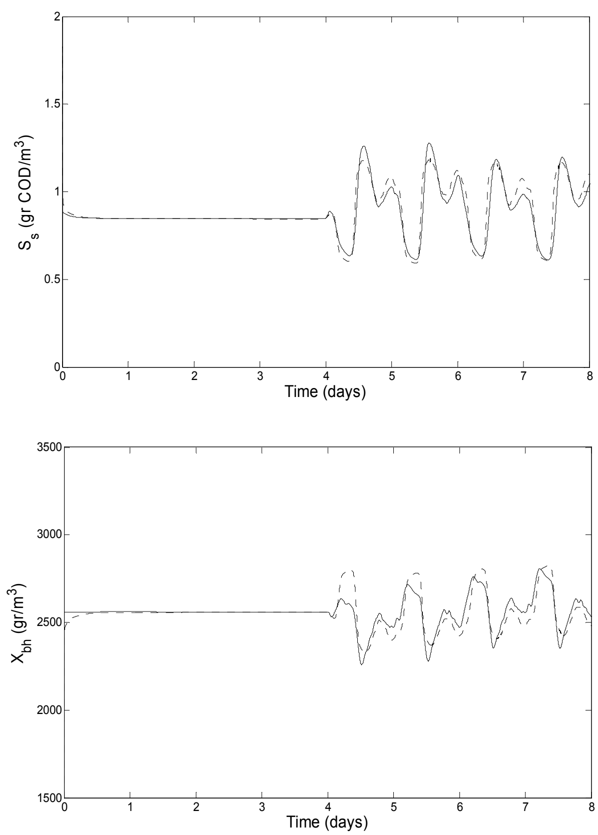



Fig.5. Fast biodegradable substrate and heterotropic biomass concentrations (solid line) and their respective estimates (dashed line).

## IV. Control Strategy

### A. L/A Structure

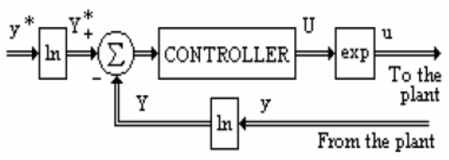



Fig.6. L/A controller.

This structure is discussed in (Lakrori *et al.,* 1989), and portrayed in Fig. 6; it is based on the following logarithmic transformation

$$\begin{aligned} Y(t) &= \ln y(t) \\ Y^*(t) &= \ln y^*(t) \\ U(t) &= \ln u(t) \end{aligned} \tag{21}$$

and the exponential transformation

$$\begin{aligned} y(t) &= \exp Y(t) \\ y^*(t) &= \exp Y^*(t) \\ u(t) &= \exp U(t) \end{aligned} \tag{22}$$

where $y(t)$ is the output, $y^*(t)$ is the set point, and $u(t)$ is the control action. These transformations allow to select any conventional control law and to obtain an L/A equivalent.

### B. Fuzzy Supervisory Control

The Takagi –Sugeno system is a special case of "functional fuzzy systems":

$$\begin{aligned} &\text{If } u_i \text{ is } A_1^j \text{ and } u_2 \text{ is } A_2^j \text{ and, ..., and } u_n \text{ is } A_n^j \\ &\text{then } b_i = g_i(.) \end{aligned} \tag{23}$$

where "." represents the arguments of the function $g_i$ and the $b_i$ are not output membership function centers. The premise of this rule is defined with linguistic terms like for the standard fuzzy system. The consequent is different; instead of linguistic terms with an associated membership function, we use a function $b_i$=$g$(.), which does not have an associated membership function. The choice of this function depends on the application being considered. Virtually any function can be used (e.g. a linear equation, neural network mapping or another fuzzy system), which makes the functional fuzzy system very general. The functional fuzzy system can use an appropriate logical operation for representing the premise (e.g., minimum or product) and defuzzification may be obtained using

$$y = \frac{\sum_{i=1}^{R} b_i \mu_i}{\sum_{i-1}^{R} \mu_i} \tag{24}$$

where $\mu_i$ is the membership value defined as

$$\mu_i(u_1, u_2, ..., u_n) = \mu_{A_1^j}(u_1) * \mu_{A_1^k}(u_2) * ... * \mu_{A_n^l}(u_n)$$

One way to view the functional fuzzy system is as a nonlinear interpolation between the mappings that are defined by consequents of the rules. When the consequent function are dynamic system then the functional fuzzy system is named as Takagi-Sugeno one (Passino *et al.,* 1998), such as

$$b_i = g_i(.) = a_{i,0} + a_{i,1}u_1 + ... + a_{i,n}u_n$$

where $a_{i,j}$ are real numbers.

### C. Hybrid Intelligent Scheme

For this strategy, we use fast biodegradable substrate and heterotropic biomass concentrations, estimated by the proposed neural observer scheme. This strategy is based on the following reasoning: if there is an excessive amount of biomass concentration, then the suspended solids increase. If the biomass concentration is low and substrate concentration is high, influent pollution cannot be treated. Both cases degrade treated water quality. For these reasons, we propose a relation *RT*, which has to be kept constant;

$$RT = \frac{X_{bh}}{S_s} \tag{25}$$

The hybrid intelligent control is based on *RT* regulation, using the scheme shown in Fig. 7; in this scheme, a PI L/A controller is considered to control $S_o$ using as manipulated variable *kla* (aeration constant); for a detailed explanation see (Vera *et al.,* 2003)

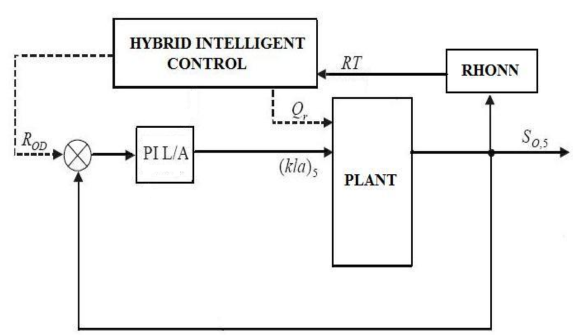



Fig.7. Hybrid intelligent control scheme.

The structure of the hybrid intelligent scheme uses a fuzzy supervisor to modify the oxygen set point ($R_{OD}$) of the L/A controller and the external recycle flow rate $Q_r$. The respective fuzzy sets are defined as

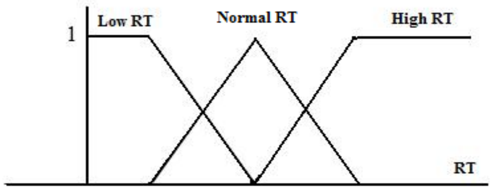



Fig.8 Input membership functions.

The respective rules are:

If $RT$ is low then $Q_r = Q_{rl}$ and $R_{OD}=R_l$

If $RT$ is normal then $Q_r = Q_{rn}$ and $R_{OD}=R_n$

If $RT$ is high then $Q_r = Q_{rh}$ and $R_{OD}=R_h$

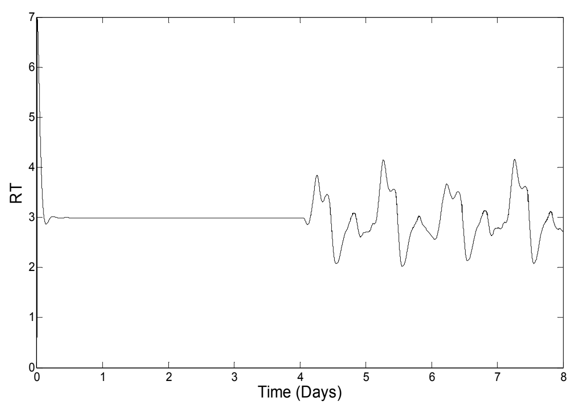



Fig.9. RT Tracking using intelligent control.

The tracking result of $RT$ is presented in Fig.9, where we can see how the hybrid system regulates the ratio $RT$ even though a constant perturbation is incepted. On the other hand, when a time variable perturbation is presented, $RT$ is kept close to the desired proportion. Furthermore, we can see the neural observer performance under control actions as is shown in Fig.10.

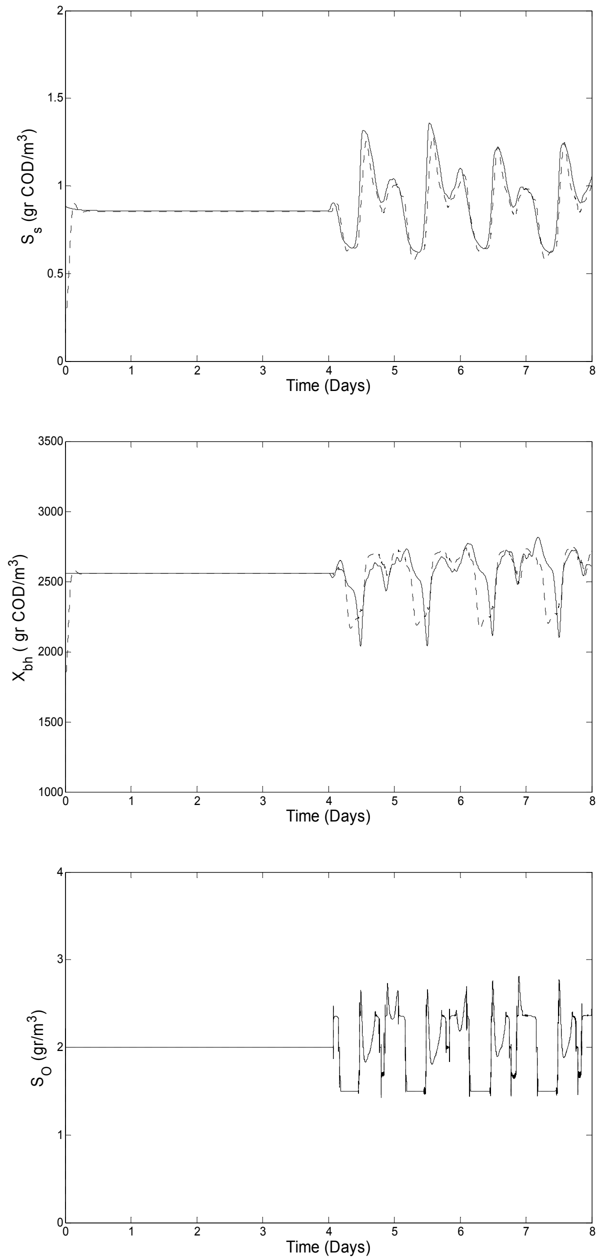



Fig.10. Fast biodegradable substrate, heterotropic biomass and oxygen dissolved concentrations (solid line) and their respective estimates (dashed line).

## V. Conclusions

In this paper, the estimation of concentrations of fast biodegradable substrate and heterotropic biomass in a WWTP has been implemented, using a RHONO and considering only on-line measurements of the dissolved oxygen. Simulations results show the effectiveness of the observer in presence of input disturbances. Additionally, the proposed fuzzy supervisor allows to regulate the substrate/biomass ratio. The whole hybrid intelligent scheme provides promising guidelines to tackle the problem of WWTP control.

## Acknowledgment

The authors thank the support of CONACYT Mexico Project 57801Y.

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